Short-time dynamics and magnetic critical behavior of two-dimensional random-bond Potts model

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Abstract

The critical behavior in the short-time dynamics for the random-bond Potts ferromagnet in two-dimensions is investigated by short-time dynamic Monte Carlo simulations. The numerical calculations show that this dynamic approach can be applied efficiently to study the scaling characteristic, which is used to estimate the critical exponents θ, β/ν and z, for the quenched disorered systems from the power-law behavior of the kth moments of magnetizations.

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1   Introduction

The understanding of the role played by quenched impurities on the nature of phase transitions is one of the significant subjects in statistical physics, and it has been a topic of substantial interest for many authors in the last two decades [1, 2, 3, 4, 5, 6, 7, 8, 9]. According to the Harris criterion[11], quenched randomness is a relevant perturbation at the second-order critical point when its specific-heat exponent α of the pure system is positive. Following the earlier work of Imry and Wortis[1] who argued that a quenched disorder could produce rounding of a *first-order* phase transition and thus induce *second-order* phase transitions, the introduction of randomness to systems undergoing a first-order phase transition is comprehensively considered. It was shown by Hui and Berker that the bond randomness can have a drastic effect on the nature of a first-order phase transition by phenomenological renormalization group arguments [2], and the feature has been placed on a firmer basis with the rigorous proof of vanishing of the latent heat [3]. Their theory was numerically checked with the Monte Carlo (MC) method by Chen, Ferrrngberg and Landau (CFL) [4, 5], where the eight-state Potts model was studied with random-bond disorders. Experimental evidence in two-dimensional systems was found that in the order-disorder phase transitions of absorbed atomic layers, the critical exponents are modified, in the addition of disorder, from the original four-state Potts model universality class in the pure case [12, 13]. On the other hand, no modification is found when the pure system belongs to the Ising Universality class [14]. The theoretical study of such kinds of disordered systems is also an active field where a resort to intensive MC simulations is often helpful [10, 15, 16, 17, 18, 19].

It is well known that the pure Potts model in two-dimensions (2D) has a second order phase transition when the number of Potts state $q \leq 4$ and is first order for $q > 4$. As the specific heat exponent α of the pure system is always positive for $q > 2$, the disorders will be the relevant perturbation for the Potts model. As a result, all the transitions are second order for all 2D q-state Potts models in the presence of quenched disorders, and the impurities have particularly strong effect for $q > 4$, even changing the order of the transitions.

In this paper, we discuss the dynamic scaling features of random-bond Potts model (RBPM) through MC simulations, to estimate the critical exponents. We consider the important questions of whether there exists an Ising-like universality class for the RBPM and how is the critical behavior affected by the introduction of disorder into the pure system [17]. The large-scale MC results by the CFL and in ref.[18] suggest that, in 2D, any random

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systems should belong to the pure Ising universality class. These results are also coherent with experiment [12]. In recent papers[7, 8], however, Cardy and Jacobsen studied the random-bond Potts models for several values of q with a different approach based on the connectivity transfer matrix (TM) formalism of Blöte and Nightingale [20] Their estimates of the critical exponents led to a continuous variation of β/ν with q, which is in sharp disaggreement with the MC results for $q = 8$ [4, 5]. We hope that the resulting critical behavior measured in this paper will play a role in settling this controversy. Furthermore we will test the *short-time dynamic* (STD) MC approach for the first time, to study the spin systems with the quenched disorder and to show its efficiency by numerical studies, which is also one of our main aims of this paper.

2 Model and Method

The Hamiltonian of q-state Potts model with quenched random interactions can be written,

$$- \beta H = \sum_{<i,j>} K_{ij}\delta_{\sigma_i\sigma_j} , \quad K_{ij} > 0, \tag{1}$$

where the spin σ can take the values $1, \cdots q$, $\beta = 1/k_B T$ the inverse temperature, δ is the Kronecker delta function, and the sum is over all nearest-neighbor pairs on a 2D lattice. The dimensionless couplings K_{ij} are selected from two positive (ferromagnetic) values of K_1 and $K_2 = rK_1$, with a strong to weak coupling ratio $r = K_2/K_1$ called as *disorder amplitude*, according to a bimodal distribution,

$$P(K) = p\delta(K - K_1) + (1 - p)\delta(K - K_2) . \tag{2}$$

When $p = 0.5$, the system is *self-dual* and the exact critical point can be determined by [21],

$$(e^{K_c} - 1)(e^{K'_c} - 1) = q . \tag{3}$$

where K_c and K'_c are the corresponding critical values of K_1 and K_2 respectively at the transition point. While $r = 1$ corresponds to the pure case and the critical point is located at $K_c = \log(1 + \sqrt{q})$ and the phase transitions are first-order for $q > 4$. With additional random-bond distribution, however, new second-order phase transitions are induced for any of q-state Potts models and the new critical points are determined according to Eq.(3) for different values of disorder amplitude r and state parameter q.

In this work we chose $q = 8$, which is known to have a strong first-order phase transition, in the hope that we would find a new second-order phase transition caused by the quenched disorder to show the effect of impurities on the *first-order* systems. The strength of the disorder was chosen for several values of r, as was done in [5, 6, 10], to check the Ising-like universality class. To minimize the number of bond configurations needed for the disorder averaging, we confined our study to the bond distributions in which there are the same number of strong and weak bonds in each of the two lattice directions. This procedure should reduce the variation between different bond configurations, with no loss of generality.

We performed our simulations by the STD method [22] on the 2D square lattices with periodic boundary conditions. This dynamic MC simulations have been successfully performed to estimate the critical temperatures T_c and the critical exponents θ, β, ν and dynamic exponent z for the 2D Ising model [23] and the 2D 3-state Potts model [24], since for both models there exist second order phase transitions. Recently this approach has also been extensively applied for the Fully Frustruated XY model and spin glass systems to study the critical scaling characteristics to estimate all the dynamic and static critical exponents [25, 26, 27].

Traditionally it was believed that universality and scaling relations can be found only in the equilibrium stage or *long-time* regime. In Ref.[30], however it was discovered that for a magnetic system in states with a very high temperature $T \gg T_c$ which is suddenly quenched to the critical temperature T_c and then evolve according to a dynamics of model A [31], there emerges a universal dynamic scaling behavior already within the short-time regime, which satisfies,

$$M^{(k)}(t, \tau, L, m_0) = b^{-k\beta/\nu} M^{(k)}(b^{-z}t, b^{1/\nu}\tau, b^{-1}L, b^{x_0}m_0), \qquad (4)$$

where $M^{(k)}$ is the kth moment of the magnetization, $\tau = (T - T_c)/T_c$ is the reduced temperature, β and ν are the well known static critical exponents and b is a scaling factor. The variable x_0, a *new independent* exponent, is the scaling dimension of initial magnetization m_0. This dynamic scaling form is generalized from finite size scaling in the equilibrium stages[32]. Importantly the scaling behavior of Eq.(4) can be applied to both dynamic exponent measurements and the estimates of the static exponents originally defined in equilibrium.

We begin our study on the evolutions of magnetization in the initial stage of the dynamic relaxation starting at very high temperature and small magnetization ($m_0 \sim 0$). For a sufficiently large lattice ($L \to \infty$), from Eq.(4) by setting $\tau = 0$, $b = t^{1/z}$, it is easy to derive

that

$$M^{(k)}(t, m_0) = t^{-k\beta/\nu z} M^{(k)}(1, t^{x_0/z} m_0). \tag{5}$$

When $k = 1$ we get the most important scaling relation on which our measurements of the critical exponent θ are based,

$$M(t) \sim m_0 t^\theta, \qquad \theta = (x_0 - \beta/\nu)/z. \tag{6}$$

As a result, the magnetization undergoes an initial increase at the critical point K_c after a microscopic time t_{mic}. This prediction is supported by a number of MC investigations which have been applied to detect all the static and dynamic critical exponents [23, 24] as well as the critical temperatures [25, 28]. The advantage of the dynamic MC simulations is that it may eliminate critical slowing down since the measurements are performed in the early time stages of the evolution where the spatial and time correlation lengths are small.

In our simulations, the time evolution of $M(t)$ is calculated through the definition

$$M(t) = \frac{1}{N}[\frac{q < M_O > -1}{q - 1}]. \tag{7}$$

Here $M_O = \max(M_1, M_2, \cdots, M_q)$ with M_i being the number of spins in the ith state among q states. $< \cdots >$ denotes the initial configuration averages over independent random number sequences, and $[\cdots]$ the disorder configuration averages over quenched random-bond distributions. $N = L^2$ is a number of spins on this square lattice and $q = 8$ is chosen.

The susceptibility plays an important role in the equibrium. Its finite size behavior is often used to determine the critical temperature and the critical exponents γ/ν and β/ν [5]. For the STD approach, the time-dependent susceptibility (the second moment of the magnetization) is also interesting and important. For the random-bond Potts model, the second moment of the magnetization is usually defined as

$$M^{(2)}(t) = \frac{1}{N}[(< M^2(t) > - < M(t) >^2)] . \tag{8}$$

To study the scaling behavior of the second moment of magnetization, we have to take the initial states of $m_0 = 0$ to start the relaxation processes. Because the spatial correlation length in the beginning of the relaxation is small compared with the lattice size L^d in the short-time regime of the dynamic evolution, the second moment behaves as $M^{(2)}(t, L) \sim L^{-d}$. Then the finite size scaling Eq.(4) induces a power-law behavior at the critical temperature,

$$M^{(2)}(t) \sim t^y, \qquad y = (d - 2\beta/\nu)/z. \tag{9}$$

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From the scaling analysis of the spatial correlation function we easily realize the non-equilibrium spatial correlation length $\xi \sim t^{1/z}$. Therefore $M^{(2)}(t) \sim \xi^{(d-2\beta/\nu)}$.

In the above considerations the dynamic relaxation process was assumed to start from a disordered state or with small magnetization m_0. Another interesting and important process is the dynamic relaxation from a completely ordered state. The initial magnatization locate exactly at its fixed point $m_0 = 1$, where scaling of the form,

$$M^{(k)}(t, \tau, L) = b^{-k\beta/\nu} M^{(k)}(b^{-z}t, b^{1/\nu}\tau, b^{-1}L), \tag{10}$$

is expected. This scaling form looks to be the same as the dynamic scaling one in the long-time regime, however, it is now assumed already valid in the macroscopic short-time regime.

For the magnetization itself, $b = t^{1/z}$ yields, for a sufficiently large lattice,

$$M(t, \tau) = t^{-\beta/\nu z} M(1, t^{-\beta/\nu z}\tau). \tag{11}$$

This leads to a power-law decay behavior of

$$M(t, \tau) = t^{-c_1}, \qquad c_1 = \beta/\nu z, \tag{12}$$

at the critical point ($\tau = 0$). The formula can be used to calculate the critical exponents β/ν and z. For a small but nonzero τ, the power-law behavior will be modified by the scaling function $M(1, t^{-\beta/\nu z}\tau)$, which has been used to determine the critical temperatures [28, 29]. Furthermore, by introducing a Binder cumulant

$$U(t, L) = \frac{M^{(2)}(t, L)}{(M(t, L))^2} - 1 \,, \tag{13}$$

a similar power-law behavior at the critical point induced from the scaling Eq.(10) shows that,

$$U(t, L) \sim t^{c_2} \,, \qquad c_2 = d/z \,, \tag{14}$$

on a large enough lattice. Here, unlike the relaxation from the disordered state, the fluctuations caused by the initial configurations are much smaller. In pratical simulations, these measurements of the critical exponents and critical temperature are better in quality than those from the realization process starting from disordered states.

Table 1: The tendency and measured values of θ as a function of the disorder amplitude r for different initial m_0 at the critical points K_c on the lattice 64^2 lattice.

m_0	0.06	0.04	0.02	0.01	θ	$K_c(r)$
$r = 2$	0.310(8)	0.338(8)	0.350(8)	0.352(7)	0.353(6)	0.920185271...
$r = 5$	0.160(6)	0.215(6)	0.252(5)	0.257(4)	0.262(4)	0.512307010...
$r = 8$	0.106(5)	0.167(4)	0.208(4)	0.218(3)	0.221(3)	0.367963156...
$r = 10$	0.090(4)	0.146(3)	0.193(3)	0.202(3)	0.203(3)	0.312655667...

3   MC Simulations and Results

As it was pointed out that the Heat-bath algorithm is more efficient than the Metropolis algorithm in the STD [24], and the universality is satisfied for different algorithms, we only perform the MC simulations with the Heat-bath algorithm at the critical points of 2D eight-state RBPM for an optimal disorder amplitude $r^* = 10$ which is located on the random fixed point regime with a largest value of central charge $c = 1.5300(5)$ [10]. Samples for averages are taken both for over 300 disorder distribution configurations and about $\sim$500 independent initial configurations on the square lattices L^2 with L up to 128. Statistical errors are simply estimated by performing three groups of averages with different random seed selectes for the initial configurations. It should be noted that, except for $M(t)$, the measurements of $M^{(2)}(t)$ and $U(t)$ are restricted to the initial states with $m_0 = 0$ or $m_0 = 1$. Importantly, it was verified that the critical exponents save the same value the same as those in the equilibrium or *long-time* stage of the relaxation [23]. Therefore we can measure these exponents based on the corresponding scaling relation in the initial stages of the relaxation.

We start our simulations to verify the power-law scaling behavior of $M(t)$ with several values of disorder amplitude at the critical points $K_c(r)$ (as shown in Table 1). The initial configurations are prepared with small magnetizations $m_0 = 0.06, 0.04, 0.02$ and exact zero states. In Fig.1, the time evolutions of the magnetization $M(t)$ versus the disorder amplitude r on a 64^2 lattice are displayed with a double-log scale. We can easily find that all the curves exhibit the power-law behavior predicted by Eq.(6). Thus θ can be estimated from the slopes of the curves. The values of θ as a function of the disorder amplitude r for small initial magnetization m_0 are presented in the Table 1.

We then set $m_0 = 0$ to measure the second moment of magnetization. The power-law

Table 2: The values of scaling exponents for the 2D $q{=}8$ RBPM with $r = 10$, measured from the scaling functions of $M(t)$, $M^{(2)}(t)$ and $U(t)$ respectively starting from both the random initial states and ordered states. Also listed are those for the 2D Ising and $q = 3$ Potts models, and the 3D Ising model [22, 24, 29].

exponent	m_0	2D RBPM	2D Ising	2D Potts	3D Ising
θ	~ 0.0	0.197(4)	0.191(1)	0.075(3)	0.108(2)
$y = (d - 2\beta/\nu)/z$		0.438(6)	0.817(7)	0.788(1)	
$c_1 = \beta/\nu z$	$= 1.0$	0.0390(6)	0.056(1)	0.065(1)	0.2533(7)
$c_2 = d/z$		0.518(9)	0.926(8)	0.934(9)	0.1462(12)
$2\beta/\nu = d - yz$		0.302(6)	0.240(36)	0.269(7)	1.034(4)
$2\beta/\nu$ (exact)			1/4	4/15	

behaviour of the second moment $M^{(2)}(t)$ is observed in Fig. 2, where the curves for different lattice sizes are plotted. Again, they present a very nice power-law increase. Values of scaling dimension $y = (d - 2\beta/\nu)/z$ determined from slopes of the curves during $t = [10, 200]$ are listed in the Table 2.

We furthermore set $m_0 = 1$ to observe the evolution of the magnetization and the Binder cumulant, both should show the power-law behavior as predicted by the Eq.(12) and Eq.(13). Their curves are plotted in the Figs. 3 and 4 respectively. The values of the scaling dimension $c_1 = \beta/\nu z$ and $c_2 = d/z$ are then estimated, as are also presented in Table 2. Now the results of y, $c_1 = \beta/\nu z$ and $c_2 = d/z$ can be used to estimate critical exponent β/ν shown in Table 2. For comparison, also listed in Table 2 are the correspoinding results of the scaling dimension for the Ising and $q = 3$ Potts models on 2D (3D) square (cubic) lattices, and in Table 3 we summarize the results of the critical exponent β/ν up to the present.

4 Summary and Conclusion

In this paper we have investigated the short-time critical dynamics of the random-bond Potts model on 2D lattices to verify whether it has a second order phase transition in the Ising-like universality class, by a STD study. Dynamic scaling haviour was found, and has been used to estimate the critical exponents θ, z and β/ν. Our main results are summarized in Table 2, which are obtained from the slopes of power-law curves of $M(t)$, $M^{(2)}(t)$ and $U(t)$ in the double-log scales by least–square fits.

Table 3:   The results of magnetic scaling exponent β/ν estimated by different methods for the 2D eight-state RBPM.

Authors	r	β/ν	Technique
CFL[4]	2	0.118(2)	MC
Cardy and Jacobsen[8]	2	0.142(4)	TM
Chatelain and Berche[9]	10	0.153(3)	MC
Picco[33]	10	0.153(1)	MC
Present work	10	0.151(3)	STD

Our work shows that for the RBPM, there exists a power-law behavior, which is the typical feature of a continuous phase transition in the STD processes. The r-dependence characteristics of the dynamic exponent θ gives evidence that their dynamic MC behavior is different from the pure Ising model, and the values of magnetic exponent β/ν in our calculation seems to be the same as that given by the TM formula [8], but not as that by the CFL [4, 5]. Furthermore we found that the values of dynamic exponent θ depend on the disorder amplitude r, and it suggests that the dynamic exponent z may also depend on the r, which would be interesting to examine for future study.

In conclusion, this study presents numerical evidence that the quenched impurities in the RBPM can induce new second-order phase transitions, but they appear not always to belong to Ising-like universality class, although the result of new critical exponent θ is the same for both the $r = 10$ RBPM and the Ising model within errorbars by present calculations. Second, as the effect of critical slowing down in the equilibrium stage for the RBPM is more severe than that for the pure systems, the cluster algorithms, up to now, have been frequently used in MC simulations of the 2D RBPM [4, 5, 9, 10, 19]. In present paper, however, we have applied the STD MC simulations which use local updating schemes for the 2D RBPM, and the fact that dynamic MC simulations can avoid the critical slowing down in the STD processes, where the spatial correlation length is still small, makes it easier to calculate the critical exponents. An important subject for further study is, for example, how the dependence of dynamic critical exponent β/ν on the state parameter q and the disorder amplitude r by a systematic simulation using the STD method in order to clarify the crossover behaviour from the random fixed point to a percolation-like limit [33]. This is being studied at present.

References

[1] Y. Imry and M. Wortis, Phys. Rev. **B9**(1979)3580.

[2] K. Hui and A.N. Berker, Phys. Rev. lett. **62**(1989)2507.

[3] M Aizenman and J. Wehr, Phys. Rev. lett. **62**(1989)2503.

[4] S. Chen, A.F. Ferrenbrerg and D.P. Landau, Phys. Rev. lett. **69**(1992)1213.

[5] S. Chen, A.F. Ferrenbrerg and D.P. Landau, Phys. Rev. **E52**(1995)1377.

[6] M. Picco, Phys. Rev. **B54**(1996)14930; Phys. Rev. lett. **79**(1997)2998.

[7] J. Cardy, J. Phys. **A29**(1996)1897.

[8] J. Cardy and J.L. Jacobsen, Phys. Rev. lett. **79**(1997)4063.

[9] C. Chatelain and B. Berche, Phys. Rev. Lett. **80**(1998)1670.

[10] C. Chatelain and B. Berche, Phys. Rev. **E58**(1998)R6899; **E60**(1999)3853.

[11] A.B. Harris, J. Phys. **C7**(1974)129.

[12] L. Schwenger, K. Buddle, C. Voges and H. Pfnür, Phys. Rev. lett. **73**(1994)296.

[13] C. Voges and H. Pfnür, Phys. Rev. **B57**(1998)3345.

[14] Ch. V. Mohan, H. Kronmüller and M. Kelsch, Phys. Rev. **B57**(1998)2701.

[15] S. Wiseman and E. Domany, Phys. Rev. **E51**(1995)3074.

[16] J.-K. Kim, Phys. Rev. **B53**(1996)3388.

[17] T. Olson and A.P. Young, Phys. Rev. **B60**(1999)3328, and references therein.

[18] M. Kardar, A.L. Stella, G, Sartoni and B. Derrida, Phys. Rev. **E52**(1995)R1269.

[19] F. Yasar, Y. Gündüc, and T. Celik, Phys. Rev. **E58**(1998)4210.

[20] H.W.J. Blöte and M.P. Nightingale, Physica **A112**(1982)405.

[21] W. Kinzel and E. Domany, Phys. Rev. **B23**(1981)3421.

[22] B. Zheng, Int. J. Mod. Phys. **B12**(1998)1419.

[23] Z.B. Li, L. Schülke, and B. Zheng, Phys. Rev. Lett. **74**(1995)3396;
Phys. Rev. **E53**(1996)2940.

[24] K. Okano, L. Schülke, K. Yamagishi and B. Zheng, Nucl. Phys. **B485 [FS]** (1997)727.

[25] H.J. Luo, L. Schülke and B. Zheng, Phys. Rev. Lett. **81**(1998)180; Phys. Rev. **E57**(1998)1327.

[26] H.J. Luo, L. Schülke and B. Zheng, accepted by Mod. Phys. Lett. **B**(cond-mat/9909325).

[27] H.P. Ying, H.J. Luo, L. Schülke, and B. Zheng, Mod. Phys. Lett. **B12**(1998)1237.

[28] L. Schülke, B. Zheng, Phys. Lett.**A204**(1995)295.

[29] A. Jaster, J. Mainville, L. Schülke and B. Zheng, J. Phys. A: Math. Gen. **32**(1999)1395.

[30] H.K. Janssen, B. Schaub, and B. Schmitmann, Z. Phys. **B73**(1989)539.

[31] P.C. Hohenberg and B.I. Halperin, Rev. Mod. Phys. **49**(1977)435.

[32] K. Binder and D.W. Heermann, *Monte Carlo Simulation in Statistical Physics* (Springer, Berlin, 1992).

[33] M. Picco, e-print cond-mat/9802092.

FIGURE CAPTIONS

FIGURE 1: The time evolution of magnetization showing the r–dependence of θ, plotted with a double-log scale on a lattice of 64×64 with $m_0 = 0.01$.

FIGURE 2: The time evolution of second moment of magnetization starting from absolute random states, plotted with a double-log scale on lattices of 32×32, 64×64 and 128×128.

FIGURE 3: The power-law decay of magnetization starting from fully ordered states, plotted with a double-log scale on lattices of 32×32, 64×64 and 128×128. The finite size effect is obvious when the lattice size $L < 64$.

FIGURE 4: The time evolution of Binder comulant starting from fully ordered states, plotted with a double-log scale on lattices of 32×32, 64×64 and 128×128.

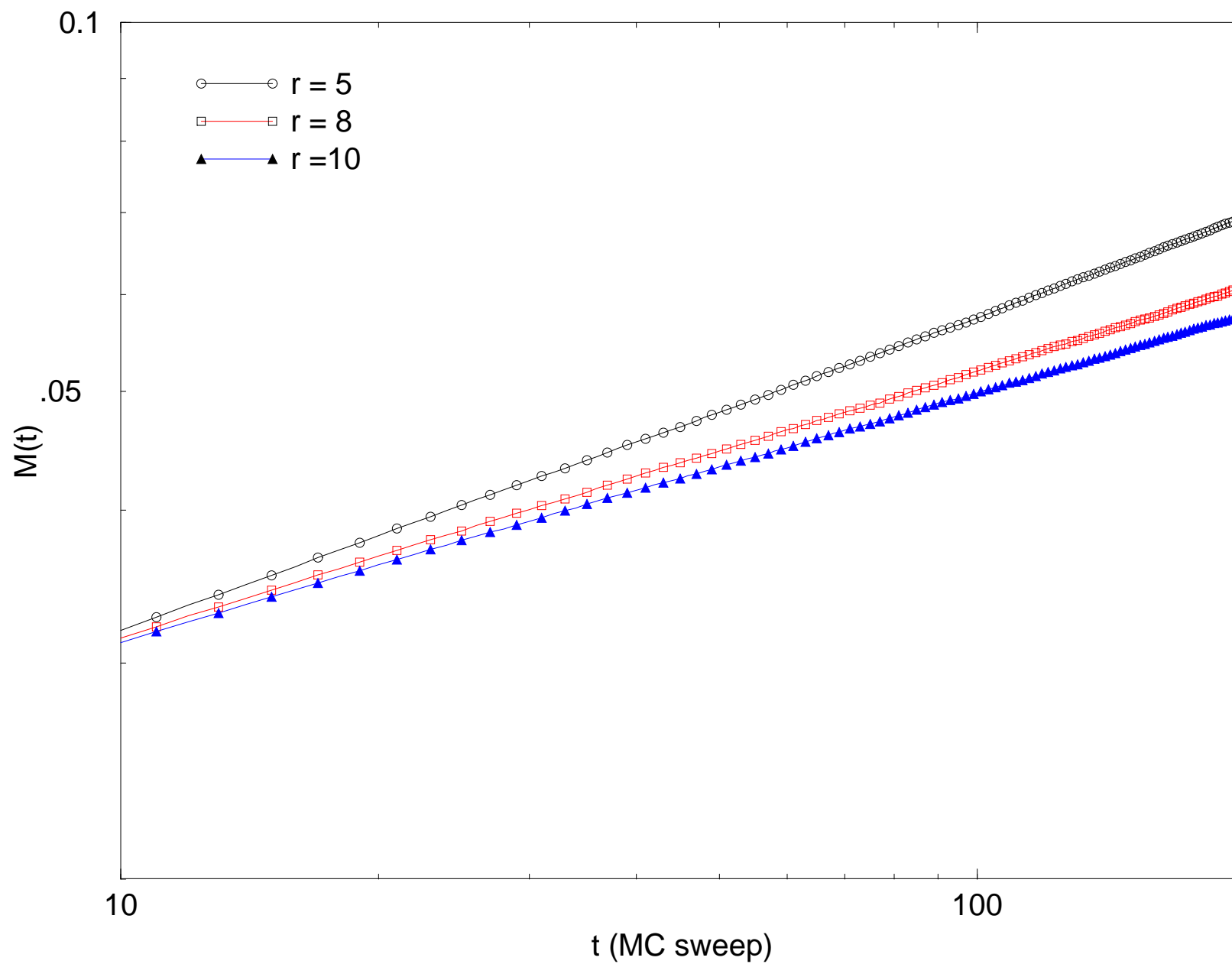Fig.1: Evolution of magnetization in a log–log scale with m_0=0.01

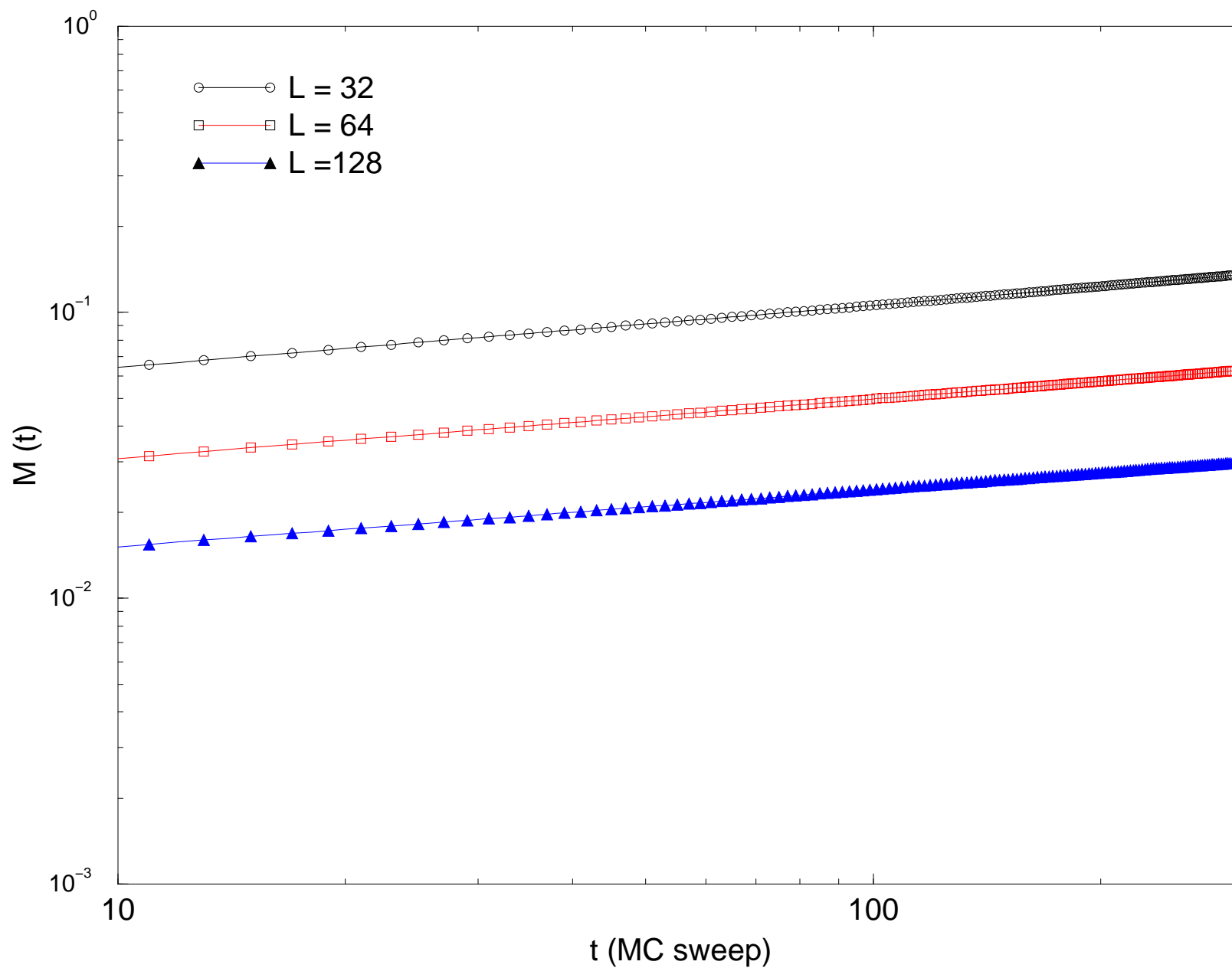Fig.2: Evolution of susceptibility from random−initial states

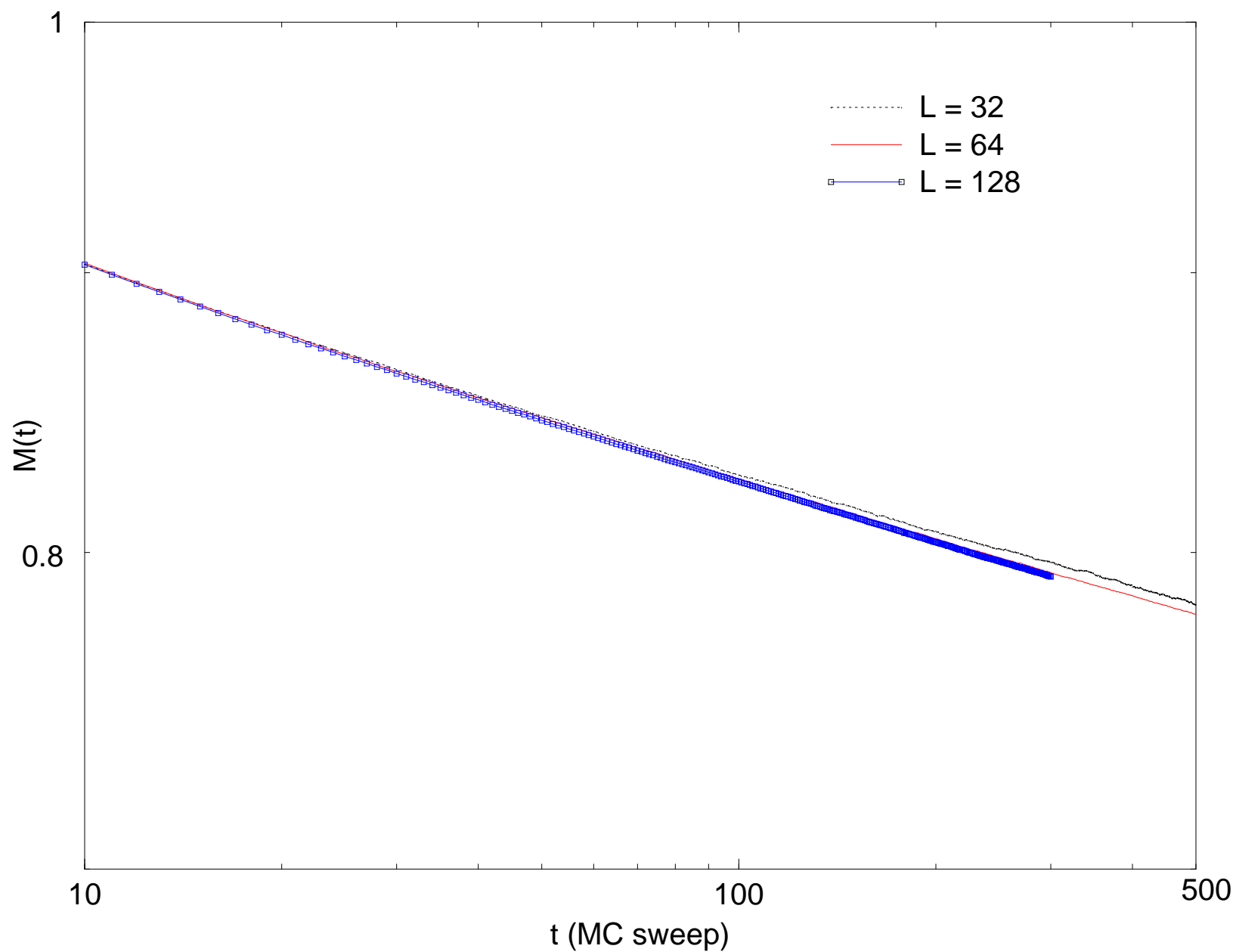Fig.3: Power−law decay of magnetization from ordered−initial states

Fig.4: Evolution of Binder cumulant from ordered−initial states